Math 250 **1.1 Review of Functions**

Objectives:

1) Functions and relations
   a. Independent and dependent variable
   b. Argument
   c. Vertical line test
   d. Domain and range: from graph, algebra, or context
      i. Find the domain and range of a function
         1. Square (and even-index) roots: $f(x) = \sqrt{\arg}$
            a. To find domain, make $\arg \geq 0$.
            b. Range is non-negative.
            c. Odd-index roots have neither of these limitations.
         2. Rational: $f(x) = \dfrac{p(x)}{r(x)}$,
            a. To find domain, exclude values of x where $r(x) = 0$.
            b. Range excludes horizontal asymptotes. (Use degrees of numerator and denominator.)
         3. Use GC to check

2) Algebra of functions
   a. Sum, difference, product, quotient
   b. Composition
      i. Find composition given component functions, graphs, or tables
      ii. Identify component functions from given composition
      iii. Domain and range

3) Lines Review
   a. Find slope of a line passing through two points
      i. Slope formula $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ [Learn GC command >frac, MATH menu!]
      ii. $m > 0$, increases; $m < 0$ decreases; $m = 0$ is horizontal (equation $y = k$)
      iii. m undefined is vertical (equation $x = h$)
   b. Write the equation of a line given a point and slope or two points
      i. Point-slope formula $y - y_1 = m(x - x_1)$
      ii. Slope-intercept form $y = mx + b$, with slope m and y-intercept $(0, b)$
      iii. General form $ax + by + c = 0$, a, b, c not fractions, $a \geq 0$
   c. Interpret slope in real-life applications
      i. Ratio – slope m has units which cancel
      ii. Rate – slope m has different units which don't cancel (e.g. miles per hour)

4) Calculate slope of secant line using the difference quotient, two forms
   a. $m_{\sec} = \dfrac{f(x+h) - f(x)}{h}$ or
   b. $m_{\sec} = \dfrac{f(x) - f(a)}{x - a}$

5) Factoring review (see separate sheet)

6) Use symmetry: Symmetry exists if you get the same equation after substituting and simplifying

   a. With respect to x-axis: (x, y) and $(x, -y)$, points above and below the x-axis are on the graph.
      i. To confirm algebraically, substitute $(x, -y)$, simplify.
      ii. This symmetry is a reflection over a line of symmetry.
      iii. Equations having this symmetry always fail the vertical line test and are not functions.
      iv. Memorable example: $x = y^2$

   b. With respect to y-axis: (x, y) and $(-x, y)$, points left and right the y-axis are both on the graph – also called an "even function"
      i. To confirm algebraically, substitute $(-x, y)$, simplify.
      ii. This symmetry is a reflection over a line of symmetry.
      iii. Memorable example: $y = x^2$
      iv. Even: (having y-axis symmetry), $f(-x) = f(x)$, points left and right of the y-axis

   c. With respect to origin: (x, y) and $(-x, -y)$, points rotated around the origin are both on the graph – also called an "odd function"
      i. To confirm algebraically, substitute $(-x, -y)$, simplify.
      ii. This symmetry is a *rotation* around a point of symmetry.
      iii. Memorable example: $y = x^3$
      iv. Odd: (having origin symmetry), $f(-x) = -f(x)$, points revolved around the origin

Practice

1) Identify the argument
   a. $f(3)$
   b. $f(x)$
   c. $f(x+h)$
2) Find the domain and range. Use GC to confirm.
   a. $f(x)=\sqrt{x^2-3x+2}$
   b. $h(x)=\frac{8}{3}x^{-1/3}-2x$
   c. A cylindrical water tower with a radius of 10 m and a height of 50 m is filled to a height of h. The volume V of water (in m^3) is given by the function $g(h)=100\pi h$
3) Factor completely
   a. $2x-12x^3+2x^2$
   b. $4a^2+20ab+25b^2$
   c. $3x^4-48$
   d. $27x^3+64y^6$
4) Using $f(x)=2x^2-1$
   a. Sketch the graph
   b. Draw a secant line through $f(-1)$ and $f(2)$
   c. Find the equation of the secant line drawn.
5) For each function, evaluate $\frac{f(x+h)-f(x)}{h}$ and $\frac{f(x)-f(a)}{x-a}$
   a. $f(x)=2x^2-1$
   b. $g(x)=\frac{5x-2}{x+3}$
   c. $k(x)=\sqrt{3x+7}$
6) $y=\sqrt{25-x^2}$
   a. Identify three functions which were composed, and in what order
   b. Test for symmetry
7) Test for symmetry:
   a. $xy-\sqrt{4-x^2}=0$
   b. $x=|y|$
8) Determine if $f(x)=4x^2-x$ is even, odd, or neither.

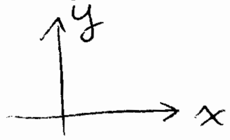
Math 250 1.1

Functions and Relations

A relation is any equation relating two variables, without restriction.

A relation of y and x where y is a function of x is such that for each x value, there is at most one y-value.

A function $y(x)$, when graphed with x on the horizontal axis



will pass the vertical line test (VLT), meaning that any imaginary vertical line intersects the graph at most once.

The notation $f(x)$, pronounced "f of x" assures us of a function, and should not be used when y is not a function of x.

If $y=f(x)$ then x is called the independent variable, and y is called the dependent variable. Need for GC Table!

The independent variable can be replaced by an expression, which is called the argument.

Practice

① Identify the argument

a) $f(3)$ The argument is $\boxed{3}$

b) $f(x)$ The argument is $\boxed{x}$.

c) $f(x+h)$ The argument is $\boxed{x+h}$.

The domain D of a function (or relation) is the set of all real values of the independent variable, determined:

a) algebraically, through recognition of specific operations

b) graphically, by projecting all points to the x-axis

c) contextually, if a word problem does not make sense for some values

The range R of a function (or relation) is the set of all real values of the dependent variable, determined

a) algebraically by recognition

b) graphically, by projecting all points to the y-axis

c) contextually

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Practice

(2) Find the domain and range. Use GC to confirm.

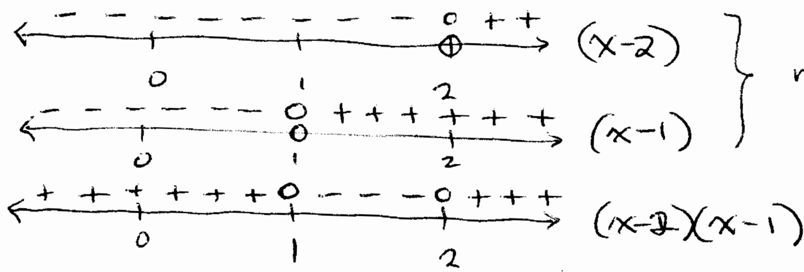
a) $f(x) = \sqrt{x^2 - 3x + 2}$

Resulting y-coordinates must be real (no i) so the argument of the square root must not be negative, i.e. ≥ 0.

Domain: $x^2 - 3x + 2 \geq 0$

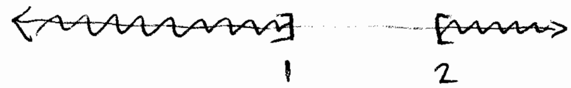
Solve polynomial inequality

$(x-2)(x-1) \geq 0$



multiply signs
$(-)(-) = (+)$
$(-)(+) = (-)$

Solution



$$\boxed{\begin{array}{l}\text{Domain } (-\infty, 1] \cup [2, \infty) \\ \text{or } \{x : x \leq 1 \text{ or } x \geq 2\}\end{array}}$$

interval notation

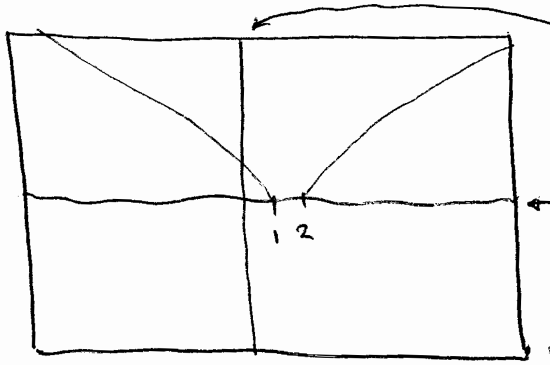
set notation

Range: Square root function $g(x) = \sqrt{x}$ always results in a number which is 0, positive (or imaginary)

$$\boxed{\begin{array}{l}\text{Range } [0, \infty) \\ \text{or } \{y : y \geq 0\}\end{array}}$$

Check by GC: $\boxed{y=}$ $y_1 = \sqrt{}(x^2 - 3x + 2)$

$\boxed{\text{ZOOM}}$ $\boxed{6}$ standard window



Range along y values, no graph below $y=0$

Domain along x values, no graph between 1 and 2

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Practice continued

(2) continued

b) $h(x) = \frac{8}{3}x^{-1/3} - 2x$

Rewrite $x^{-1/3} = \frac{1}{x^{1/3}} = \frac{1}{\sqrt[3]{x}}$

negative exponent to denominator
fraction exponent to radical.

$$h(x) = \frac{8}{3\sqrt[3]{x}} - 2x$$

This term is linear (degree 1)
Alone, its domain and range are unrestricted

The cube root has no restrictions, either:

ex: $\sqrt[3]{-8} = -2$

$\sqrt[3]{0} = 0$.

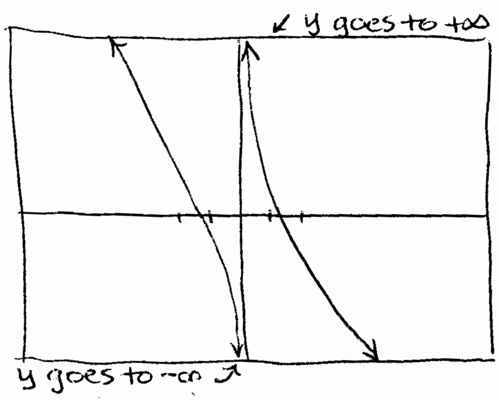
However division by a variable causes $\div 0$ when the variable is 0.

So $x \neq 0$.

Domain $(-\infty, 0) \cup (0, \infty)$
or $\{x \mid x \neq 0\}$

Use GC to determine range

$\boxed{y=}$ $y_1 = 8/3 * x$ ^ $(-1/3) - 2x$



There appears to be a vertical asymptote at $x = 0$

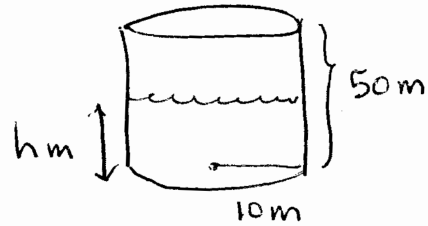
Range $(-\infty, \infty)$
or $\{y : y \in \mathbb{R}\}$

Practice continued

(2) con't

c) A cylindrical water tower with radius 10 m and height 50 m is filled to a height of h. The volume V of water in the tank (in m^3) is given by the function $g(h) = 100\pi h^2$

Aside: Do you believe that this function is correct?



volume of cylinder
$V = \pi r^2 h$

radius of water = 10 m
height of water = h

$V = \pi (10)^2 \cdot h$

$V = \pi \cdot 100 \cdot h$

$V(h) = 100\pi h$

Domain: valid values of independent variable, h.

lowest water level: 0 m

highest water level: 50 m.

$\boxed{\text{Domain } [0, 50] \text{ or } \{h : 0 \le h \le 50\}}$ in meters

Range: valid values of dependent variable, V.

lowest water level $h = 0$ gives $V = 100 \cdot \pi \cdot 0 = 0\ m^3$

highest water level $h = 50$ gives $V = 100\pi \cdot 50$
$= 5000\pi\ m^3$

$\boxed{\text{Range } [0, 5000\pi] \text{ or } \{V : 0 \le V \le 5000\pi\}}$ in m^3.

Math 250

Process for Factoring

Step 0: Arrange the terms in standard form, descending from the leading (highest-degree) term first.
(If there is more than one variable, choose a variable and arrange in descending order by that variable.)

Step 1: Factor out the greatest common factor from all terms.

Step 2: Count the terms.
(Terms are separated by add or subtract symbols, except when the addition or subtraction symbol is already inside parentheses.)

Step 3: If you have 2 terms, factor it by its pattern:
3a: Sum of squares: $a^2 + b^2$ is prime.
3b: Difference of squares: $a^2 - b^2 = (a-b)(a+b)$
3c: Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
3d: Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
[In 3c and 3d the acronym SOAP can be used to remember the three signs in the factors:
Same – **O**pposite – **A**lways **P**ositive]

Step 4: If you have 3 terms, determine which of the following applies:

4a: Perfect Square Trinomial (sum): $a^2 + 2ab + b^2 = (a+b)^2$

4b: Perfect Square Trinomial (difference): $a^2 - 2ab + b^2 = (a-b)^2$

4c: Leading coefficient 1: $x^2 + bx + c$; Find two numbers that multiply to c and add to b using guess-and-check or magic X.

4d: Leading coefficient not 1: $ax^2 + bx + c$;
Use guess-and-check by finding numbers for the first terms that multiply to a and numbers for the second terms that multiply to c, or
Use the "double magic X" by finding two numbers that multiply to the product ac and add to b, then use these to rewrite the middle term and factor by grouping.

4e: If the expression is quadratic in form, $a(garbage)^2 + b(garbage) + c$,
Substitute u=*garbage* to get a true quadratic, factor using u and one of the methods 4a-4d, then replace u by *garbage*, simplify inside parentheses. Check for greatest common factor.

Step 5: If you have 4 terms, factor by grouping.

4a: Two groups of two terms results in two binomial factors.

4b: [Less common]: group three terms to make a perfect square trinomial minus a constant, then factor as a difference of squares.

Step 6: Check each factor to see if it can be factored. Continue factoring until every factor is prime.
(When you are done, you should have one term with all add and subtract signs inside parentheses.)

Step 7: When in doubt, multiply your result. You should get your original expression (or a simplified version of it). Factoring is the opposite of multiplying.

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Practice continued

3 Factor completely

a) $2x - 12x^3 + 2x^2$

$= -12x^3 + 2x^2 + 2x$ write in standard form (decreasing exponents)

$= -2x(\underbrace{6x^2 - x - 1})$ factor GCF, including negative because leading coefficient -12 was negative.

focus on trinomial. → leading coefficient of the trinomial is not 1.

$6(-1) \leftarrow a \cdot c \leftarrow$ seek two #s that multiply to -6 but add to -1

-6

-3 2

$-1 \leftarrow b \leftarrow$

Trinomial:

$6x^2 - x - 1$

$= \underbrace{6x^2 - 3x}_{\text{GCF } 3x} + \underbrace{2x - 1}_{\text{GCF } +1}$ rewrite middle term and group. (This process can be replaced by guess-and-check.)

$= 3x(2x-1) + 1(2x-1)$ ← GCF $(2x-1)$

$= (2x-1)(3x+1)$

$= \boxed{-2x(2x-1)(3x+1)}$ Rewrite with GCF

b) $4a^2 + 20ab + 25b^2$

↑ ↑ notice that first and last terms (descending by variable a) are perfect squares.

$\sqrt{4a^2} \to 2a$ $\sqrt{25b^2} \to 5b$

$= \boxed{(2a+5b)^2}$

check by FOIL $(2a+5b)(2a+5b)$

$= 4a^2 + 10ab + 10ab + 25b^2$

$= 4a^2 + 20ab + 25b^2$ ✓

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Practice continued

3 cont

c) $3x^4 - 48$

$= 3(x^4 - 16)$

GCF 3

2 terms, even/no exp ⇒ difference of squares

subtracted

$a^2 - b^2 = (a+b)(a-b)$

$\sqrt{x^4} \to x^2$

$\sqrt{16} \to 4$

$= 3(x^2+4)(x^2-4)$

$x^2+4 \Rightarrow$ sum of squares is prime

$= \boxed{3(x^2+4)(x-2)(x+2)}$

$x^2-4 \Rightarrow$ difference of squares can be factored

d) $27x^3 + 64y^6$

no GCF

exponents 3 and 6 are both multiples of 3.

Sum of cubes

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$\sqrt[3]{27x^3} \to 3x$ is a in formula

$\sqrt[3]{64y^6} \to 4y^2$ is b in formula

$= (3x+4y^2)((3x)^2 - (3x)(4y^2) + (4y^2)^2)$

S — same sign

O — opposite sign

AP — always positive

⇒ Use SOAP to remember signs

$= \boxed{(3x+4y^2)(9x^2 - 12xy^2 + 16y^4)}$

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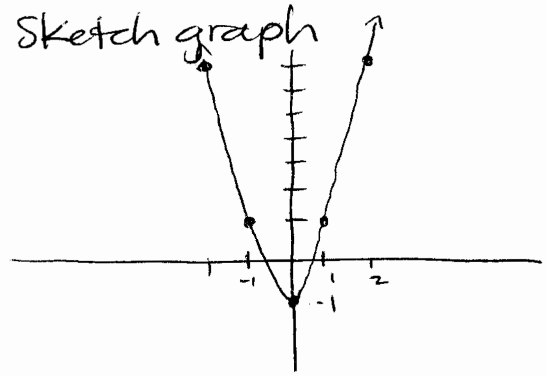
Practice continued

(4) $f(x) = 2x^2 - 1$

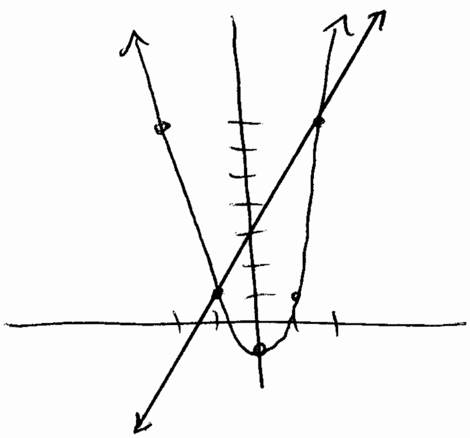
a) Sketch graph



parabola upward

x	y
-2	7
-1	1
0	-1
1	1
2	7

b) Draw secant line through $f(-1)$ and $f(2)$.



"secant" comes from Latin "secare" meaning "to cut".

$f(-1) = 2(-1)^2 - 1 = 1 \quad (-1,1)$

$f(2) = 2(2)^2 - 1 = 7 \quad (2,7)$

Draw a line connecting $(-1,1)$ and $(2,7)$

c) Find equation of secant line.

$$m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{2-(-1)} = \frac{6}{3} = 2$$

point-slope method:

$y - y_1 = m(x - x_1)$

$y - y_1 = 2(x - x_1)$

$y - 1 = 2(x - (-1))$

$y - 1 = 2(x+1)$

$y = 2x + 2 + 1$

$\boxed{y = 2x+3}$

slope-intercept method:

$y = mx + b$

$y = 2x + b$

$1 = 2(-1) + b$

$1 = -2 + b$

$3 = b$

$\boxed{y = 2x+3}$

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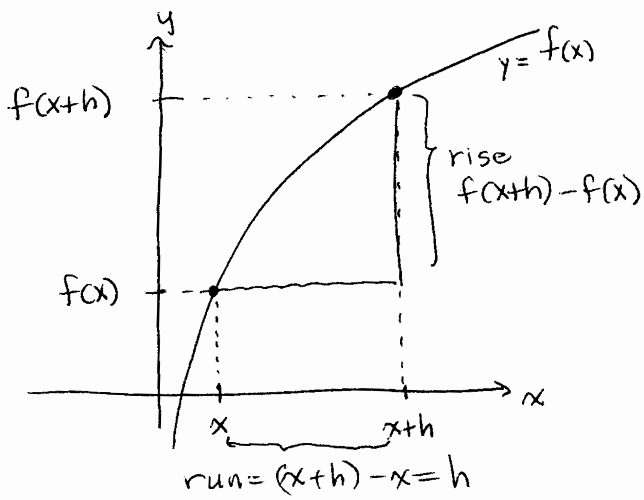
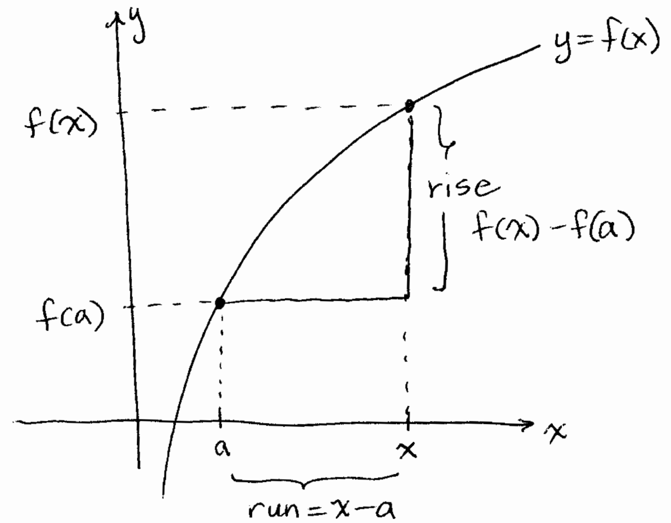
Practice continued

⑤ For each function evaluate $\dfrac{f(x+h)-f(x)}{h}$ and $\dfrac{f(x)-f(a)}{x-a}$.

<u>Notice</u>: Instructions do not say "simplify", but this <u>is</u> expected.
These expressions are called <u>difference quotients</u>

- "difference" → subtraction in numerator
- "quotient" → division by h or $(x-a)$.
- arise from finding the slope of a line connecting 2 pts
- absolutely essential in the study of calculus
- both are expressions for the slope of a secant line





$$m_{sec} = \frac{rise}{run} = \frac{f(x+h)-f(x)}{h}$$

$$m_{sec} = \frac{rise}{run} = \frac{f(x)-f(a)}{x-a}$$

a. $f(x) = 4x^2 - 1$

<u>step 1</u>: Evaluate $f(x+h)$ by replacing x by $(x+h)$, using

$$f(x+h) = 4(x+h)^2 - 1$$

$$= 4(x^2+2xh+h^2) - 1 \qquad \text{simplify FOIL}$$

$$= 4x^2 + 8xh + 4h^2 - 1$$

<u>step 2</u>: Substitute into formula for difference quotient

$$\frac{f(x+h)-f(x)}{h} = \frac{4x^2+8xh+4h^2-1-(4x^2-1)}{h}$$

parentheses on $f(x)$ are crucial

$$= \frac{4x^2+8xh+4h^2-1-4x^2+1}{h}$$

dist neg.

$$= \frac{8xh + 4h^2}{h}$$

combine like terms

* when simplifying the h version of a difference quotient, EVERY term that doesn't have h must cancel out.

If it doesn't, stop and check your work.

$$= \frac{h(8x + 4h)}{h}$$

Factor out h

$$= \boxed{8x + 4h}$$

Cancel h.

* Note to those who've had some Calculus before: there is no limit in this question ... yet. So $4h$ is part of the answer.

$f(x) = 4x^2 - 1$

$$\frac{f(x) - f(a)}{x - a} = \frac{4x^2 - 1 - (4a^2 - 1)}{x - a}$$

CAUTION
$x \neq a$

$$= \frac{4x^2 - 1 - 4a^2 + 1}{x - a}$$

substitute $f(x)$, $f(a)$
use parentheses
distribute
combine

$$= \frac{4x^2 - 4a^2}{x - a}$$

* when simplifying this version of a difference quotient, $(x - a)$ must cancel out.

$$= \frac{4(x^2 - a^2)}{(x - a)}$$

Fully factor numerator
GCF 4
difference of squares

$$= \frac{4\cancel{(x - a)}(x + a)}{\cancel{(x - a)}}$$

divide out common factor

$$= \boxed{4(x + a)}$$

or $\boxed{4x + 4a}$

b. $g(x) = \dfrac{5x-2}{x+3}$

$$g(x+h) = \frac{5(x+h)-2}{(x+h)+3}$$

* CAUTION * replace both x's

$$= \frac{5x+5h-2}{x+h+3}$$

$$\frac{g(x+h)-g(x)}{h} = \frac{\left(\dfrac{5x+5h-2}{x+h+3} - \dfrac{5x-2}{x+3}\right) \text{LCD}}{\left(\dfrac{h}{1}\right) \text{LCD}}$$

This is a complex fraction
a fraction within a fraction

LCD of all denom
$(x+h+3)(x+3)$
mult all terms, top and bottom by LCD $\rightarrow \dfrac{LCD}{LCD} = 1$

$$= \frac{\dfrac{(5x+5h-2)}{\cancel{(x+h+3)}} \cdot \cancel{(x+h+3)}(x+3) - \dfrac{(5x-2)}{\cancel{(x+3)}}(x+h+3)\cancel{(x+3)}}{\dfrac{h}{1} \cdot (x+h+3)(x+3)}$$

$$= \frac{(5x+5h-2)(x+3) - (5x-2)(x+h+3)}{h(x+h+3)(x+3)}$$

* CAUTION Don't distribute h! We want it to cancel.

$$= \frac{5x^2+15x+5xh+15h-2x-6-(5x^2+5xh+15x-2x-2h-6)}{h(x+h+3)(x+3)}$$

FOIL
use ( )
combine
dist neg

$$= \frac{5x^2+13x+5xh+15h-6-(5x^2+5xh+13x-2h-6)}{h(x+h+3)(x+3)}$$

$$= \frac{\cancel{5x^2}+\cancel{13x}+\cancel{5xh}+15h-\cancel{6}-\cancel{5x^2}-\cancel{5xh}-\cancel{13x}+2h+\cancel{6}}{h(x+h+3)(x+3)}$$

combine
check - all terms w/ h are gone YES!!

$$= \frac{17h}{h(x+h+3)(x+3)}$$

combine

$$= \frac{\cancel{h}(17)}{\cancel{h}(x+h+3)(x+3)}$$

$$= \boxed{\frac{17}{(x+h+3)(x+3)}}$$

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$$\frac{g(x)-g(a)}{x-a} = \frac{\frac{5x-2}{x+3} - \frac{5a-2}{a+3}}{\frac{x-a}{1}}$$

another complex fraction

LCD $(x+3)(\quad +3)$

$$= \frac{\frac{(5x-2)\cancel{(x+3)}(a+3)}{\cancel{(x+3)}} - \frac{(5a-2)(x+3)\cancel{(a+3)}}{\cancel{(a+3)}}}{\frac{(x-x_0)}{1}(x+3)(x_0+3)}$$

$$= \frac{5xa+15x-2a-6-(5xa+15a-2x-6)}{(x-a)(x+3)(a+3)}$$

FOIL use ( )!

$$= \frac{\cancel{5xa}+15x-2a-\cancel{6}-\cancel{5xa}-15a+2x+\cancel{6}}{(x-a)(x+3)(a+3)}$$

$$= \frac{17x-17a}{(x-a)(x+3)(a+3)}$$

factor GCF

$$= \frac{17\cancel{(x-a)}}{\cancel{(x-a)}(x+3)(a+3)}$$

$$= \boxed{\frac{17}{(x+3)(a+3)}}$$

c. $k(x) = \sqrt{3x+7}$

$$\frac{k(x+h)-k(x)}{h} = \frac{\sqrt{3(x+h)+7} - \sqrt{3x+7}}{h}$$

substitute

$$= \frac{\sqrt{3x+3h+7} - \sqrt{3x+7}}{h}$$

Must have h cancel out ???

SNEAKY THING TO REMEMBER
Rationalize the numerator.

$$= \frac{(\sqrt{3x+3h+7} - \sqrt{3x+7})}{h} \cdot \frac{(\sqrt{3x+3h+7} + \sqrt{3x+7})}{(\sqrt{3x+3h+7} + \sqrt{3x+7})}$$

- use conjugate
- use same top & bottom, mult b

$$= \frac{3x+3h+7-(3x+7)}{h(\sqrt{3x+3h+7} + \sqrt{3x+7})}$$

Foil difference of squares

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$$= \frac{3x+3h+7-3x-7}{h\left(\sqrt{3x+3h+7}+\sqrt{3x+7}\right)}$$

combine

$$= \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3x+3h+7}+\sqrt{3x+7}\right)}$$

$$= \boxed{\frac{3}{\sqrt{3x+3h+7}+\sqrt{3x+7}}}$$

$$\frac{k(x)-k(a)}{x-a}$$

$$= \frac{\sqrt{3x+7}-\sqrt{3a+7}}{x-a}$$

same trick — rationalize the numerator

$$= \frac{\left(\sqrt{3x+7}-\sqrt{3a+7}\right)}{(x-a)}\frac{\left(\sqrt{3x+7}+\sqrt{3a+7}\right)}{\left(\sqrt{3x+7}+\sqrt{3a+7}\right)}$$

$$= \frac{3x+7-(3a+7)}{(x-a)\left(\sqrt{3x+7}+\sqrt{3a+7}\right)}$$

$$= \frac{3x+\cancel{7}-3a-\cancel{7}}{(x-a)\left(\sqrt{3x+7}+\sqrt{3a+7}\right)}$$

$$= \frac{3x-3}{(x-a)\left(\sqrt{3x+7}+\sqrt{3a+7}\right)}$$

$$= \frac{3\cancel{(x-a)}}{\cancel{(x-a)}\left(\sqrt{3x+7}+\sqrt{3a+7}\right)}$$

$$= \boxed{\frac{3}{\sqrt{3x+7}+\sqrt{3a+7}}}$$

Math 250 1.1

Practice continued

⑥ $y = \sqrt{25-x^2}$

a) Identify three functions which were composed, and in what order.

Imagine evaluating when $x=3$

first	$x^2 \to 3^2$	square	$f(x) = x^2$
second	$25-3^2$	subt from 25	$g(x) = 25-x$
third	$\sqrt{25-3^2}$	sq root	$h(x) = \sqrt{x}$

want $25-x^2$ $\longrightarrow$ put x^2 into $25-x$

$f(x)$ $g(x)$ $\longrightarrow$ $g(f(x))$

check $g(f(x)) = 25 - f(x)$

$= 25 - x^2$ ✓

want $\sqrt{\text{result}}$ $\longrightarrow$ put previous result into $\sqrt{x}$

$g(f(x))$ $h(x) \to h(g(f(x)))$

check $h(g(f(x))) = \sqrt{g(f(x))}$

$= \sqrt{25-x^2}$

The three functions (could use any names)

are $f(x) = x^2$

$g(x) = 25-x$

$h(x) = \sqrt{x}$

so that $y = \sqrt{25-x^2} = h(g(f(x)))$

b) Test for symmetry. ← CAUTION: This instruction means use Algebra Not GC!

x-axis: (x,y) and $(x,-y)$ both on graph

$-y = \sqrt{25-x^2}$ $\boxed{\text{NO}}$

y-axis: (x,y) and $(-x,y)$ both on graph

$y = \sqrt{25-(-x)^2} = \sqrt{25-x^2}$ $\boxed{\text{YES}}$

origin: (x,y) and $(-x,-y)$ both on graph

$-y = \sqrt{25-(-x)^2} = \sqrt{25-x^2}$ $\boxed{\text{NO}}$

M250 1.1

7 Test for symmetry.

a) $xy - \sqrt{4-x^2} = 0$

x-axis: (x,y) and $(x,-y)$

$x(-y) - \sqrt{4-x^2} = 0$

$-xy - \sqrt{4-x^2} = 0$ $\boxed{\text{NO}}$

If you get the same equation, there's symmetry.

y-axis: (x,y) and $(-x,y)$

$(-x)y - \sqrt{4-(-x)^2} = 0$

$-xy - \sqrt{4-x^2} = 0$ $\boxed{\text{NO}}$

origin: (x,y) and $(-x,-y)$

$(-x)(-y) - \sqrt{4-(-x)^2} = 0$

$xy - \sqrt{4-x^2} = 0$ $\boxed{\text{YES}}$

b) $x = |y|$

x-axis:

$x = |-y|$

$x = |y|$ $\boxed{\text{YES}}$

y-axis

$-x = |y|$ $\boxed{\text{NO}}$

origin:

$-x = |-y|$

$-x = |y|$ $\boxed{\text{NO}}$

8 Determine if $f(x) = 4x^2 - x$ is odd, even, neither.

$f(-x) = 4(-x)^2 - (-x)$

$= 4x^2 + x \neq f(x)$

$\neq -f(x)$

$\boxed{\text{NEITHER}}$